LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIFTH SEMESTER – APRIL 2010

ST 5500 - ESTIMATION THEORY

Date & Time: 24/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

PART – A

Answer ALL the questions **(10x2=20 marks)**

1. Define unbiasedness of an estimator. Give an example.
2. If T(x) is an unbiased estimator for θ, show that $T^2(x)$ is biased for θ^2 .
3. Bring out the importance of Lehman-Scheffe Theorem.
4. Define Sufficient statistic. Give an example.
5. List out any two large sample properties of ML estimator.
6. Briefly explain the method of moments of estimating the parameters.
7. Define risk function associated with a decision function. How is it different from the variance of an estimator?
8. Explain the terms: prior and posterior probability distributions.
9. Write down the normal equations associated with a simple regression model.
10. State the Gauss - Markoff model and explain its components.

PART - B

Answer any FIVE questions **(5x8=40 marks)**

11. Let $(X_1, X_2, X_3,\ldots,X_n)$ be a random sample of size n from Normal population with unknown mean μ and variance1. Obtain an unbiased estimator for the population mean and examine whether it is consistent.

12. Let $(X_1, X_2, X_3,\ldots,X_n)$ be a random sample from Poisson population with meanλ. Examine the completeness of $T(x) = \Sigma X_i$. Suggest a UMVUE for λ.

13. State and prove Factorization theorem on sufficient statistics in one parameter discrete case.

14. Derive the moment estimators of the parameters of a two parameter gamma distribution.

15. Explain the method of minimum Chi-square estimation

16. Let $(X_1, X_2, X_3,\ldots,X_n)$ be a random sample from U(a, b), $0 < a < b$. Examine the existence of ML estimators for a and b.

17. Establish a necessary and sufficient condition for a linear parametric function to be estimable.

18. Let ($X_1, X_2, X_3, \ldots, X_n$) be a random sample from N (μ, σ^2). Obtain an unbiased estimator for the population variance. Examine whether its variance attains Cramer – Rao lower bound.

PART - C

Answer any TWO questions **(2x20=40 marks)**

19. (a) Establish Chapman – Robbins Inequality and mention its importance.

(b) Let ($X_1, X_2, X_3 \ldots X_n$) be a random sample from a population whose pdf is

$$f(x,\theta) = \begin{cases} \theta e^{-(x-\theta)} & if\ x > \theta \\ 0 & otherwise \end{cases} .$$

Examine the unbiasedness and consistency of the sample minimum $X_{(1)}$.

20. a) State and prove Rao Blackwell theorem. What is its importance?

(b) Let ($X_1, X_2, X_3 \ldots X_n$) be a random sample from U(0, θ), θ>0. Examine whether the sample maximum is a complete sufficient statistic.

21. (a) Explain the method of modified minimum Chi-square estimation.

(b) Let ($X_1, X_2, X_3 \ldots X_n$) be a random sample from Bernoulli distribution with parameter p. Obtain the Bayesian estimators of mean p and variance p(1-p) by taking a suitable prior distribution.

22. (a) Obtain the method of least square estimation in a three parameter regular case.

(b) Samples of sizes n_1 and n_2 are drawn from two populations with means m_1 and m_2 and with common variance σ^2. Find the BLUE of $l_1 m_1 + l_2 m_2$.

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